Use of Complex Fuzzy Matrices in the Quality Analysis of Drinking Water

Ms. Divya Mol K T
Research scholar
Research and Post Graduate Department of Mathematics
St. Thomas College, Palai, India
e-mail id: divyaa.kt.1991@gmail.com
Dr.Lovelymol Sebastian
Research Supervisor
Research and Post Graduate Department of Mathematics
St. Thomas College Palai. Kerala. India
Corresponding author mail id: divyaa.kt.1991@gmail.com

Abstract. In this paper we use the concept of complex fuzzy matrices and some properties of complex fuzzy matrices and data defined in (Beaula et al., 2017; Millika et al., 2017). we developed a new method for drinking water quality analysis. We apply the concept of relation matrices in (Geetha et al., 2017; Usha et al., 2017) and modified algorithm in (Beaula et al., 2017; Millika et al., 2017) for the quality analysis of drinking water from different sources. An example is illustrated to verify the developed procedure.

Keywords: -Complex fuzzy sets; Complex fuzzy matrices; Relation matrix; Comparison matrix; Water Quality Analysis.
2010 Mathematics Subject Classification: **-** 15B15; 03E72

INTRODUCTION

Zadeh (Zadeh, 1965) was the founder of the concept of fuzzy set theory about 1965. The most important and interesting areas of applications of the theory of fuzzy set are the field of medicine and treatment. Fuzzy matrices are now very rich topic in modeling situations including uncertainty occurred in science, treatments, medical diagnosis, automata etc.

Fuzzy matrices were introduced by Thomson (Thomson, 1997).in 1977 and these concepts was developed by Kim and Roush. K. T Atanassov was the founder of the notion intuitionistic fuzzy sets, which providing a flexible model to elaborate uncertainty and vagueness in decision making problems. The concept of fuzzy matrices is used in almost all the branches of science. Fuzzy matrices are a better implement for modelling different problems occurring in uncertain situations in different fields of science like computer science, robotics, medical science, artificial intelligence and may others.

In 2002, Ramot*et al.*,(Ramot*et al*., 2002; Milo *et al*., 2002; Friedman *et al*., 2002; Kandel *et al*., 2002) defined the Complex Fuzzy sets as a generalization of fuzzy sets whose co domain is not restricted to $ [0, 1] $ but it is expanded on the unit disc in the complex plane (the set of all complex numbers with modulus less than or equal to 1).

Ramot used the idea of complex degree of membership in polar coordinating, where the amplitude is the degree of an object of the Complex Fuzzy Set and the role of phase is to add information which is related to spatial or temporal periodicity of the specific fuzzy set. In 2015, Zhi- QuigZaho and Shong-Quan Ma, were introduced the concept of complex fuzzy matrices.

Water has always been an important and life sustaining drink to all the human beings. Water is the most essential substance for the existence and survival of all the organisms. Availability of fresh and clean water for human use is one of the most important issues. Water quality of any specific area or from a particular source can be studied using some parameters, like physical, chemical and biological.

There are several studies on the quality of drinking water and combining these studies with analysis based on various parameters will provide clearer and accurate information about the quality of drinking water. These results and studies will enable the analyst to adopt appropriate measures to make unsafe drinking water safe.

PRELIMINARIES

Fuzzy Sets

A fuzzy set is a pair(U, μ) where U is a non-empty set and $\mu: U \to [0,1]$ a membership function. The set U is called the universe of discourse and for each $x \in U$, the value $\mu(x)$ is called the degree of membership of x in (U, μ). Then the function μ is called the membership function of the fuzzy set (U, μ).

For a finite set, $U = \{x_1, x_2, x_3, \ldots, x_n\}$, the fuzzy set (U, μ) is often denoted by,$\{\mu(x_1)/x_1, \mu(x_2)/x_2, \mu(x_3)/x_3, .., \mu(x_n)/x_n\}$

Let $x \in U$, then x is called;

- Not included in the fuzzy set (U, μ), if $\mu(x) = 0$.
- Fully included in the fuzzy set (U, μ), if $\mu(x) = 1$.
- Partially included in the fuzzy set (U, μ), if $0 < \mu(x) < 1$.

Fuzzy Matrices

A fuzzy matrix is a matrix which has its elements from the unit interval [0,1], called fuzzy unit interval.

A fuzzy matrix A of order $m \times n$ is defined as $A = [\langle a_{ij}, a_{ij\mu}\rangle]_{m\times n}$, where $a_{ij\mu}$ is the membership value of a_{ij} in A.

For simplicity, we write A as, $A = [a_{ij\mu}]_{m\times n}$

Example:

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.7 & 0.5 \\ 0.3 & 0.8 & 0.1 & 0.6 \\ 0.6 & 0.4 & 0.9 & 0.8 \\ 0.2 & 0.7 & 0.3 & 0.4 \end{bmatrix}_{4\times 4} \quad (1)$$

Remark

When we consider (Thomson, 1997) the average sunspot number since 1800. During the minimum activity, there are few sunspots, whereas during the solar maximum activity there are many sunspots. According to Ramot, (Ramot*et al*., 2002; Milo *et al*., 2002; Friedman *et al*., 2002; Kandel *et al*., 2002) a complex fuzzy set is used to convey information related to the monthly solar activity as well as its position in the unit circle. Under this formalism of complex fuzzy sets, the position in a cycle is represented by the phase variable, which is a real function, and not a degree of membership and the solar activity for a specific month is represented by a degree of membership in a fuzzy set.

For example, (Tamir *et al*., 2011; Jin*et al*., 2002; Kandel *et al*., 2011) the fuzzy set induced by the assertion, "high solar activity". As noted, the cycle of length is 11 years; it starts from a solar minimum goes through a maximum and end at the next solar minimum. Let 0 be the starting point, let π be the solar maximum and 2π denote the next solar minimum. Suppose the traditional grade of membership of a month M_i in the set "High solar activity" is 0.4, and assume that this month is at the peak of solar activity for the cycle then the grade of membership is denoted by $0.4e^{i\pi}$.

On the other hand, Mj is characterized by the complex gradeof membership 0.60.6$e^{i\frac{\pi}{2}}$, then it means that M_j is in the increasing process of solar activity and it is medium active.

Complex Fuzzy Matrices

A complex fuzzy matrix is a matrix which has its elements from the unit ball in the complex plane.

A complex fuzzy matrix A of order $m \times n$ is defined as

$A = \left[\langle a_{ij}, a_{ij\mu}\rangle\right]_{m\times n};\ a_{ij\mu} = r_{ij}e^{i\omega_{ij}}; r_{ij} \in [0,1];\ \omega_{ij} \in [0,2\pi)$ (2)

Example: $A = \begin{bmatrix} 0.7e^{i\pi} & 0.5e^{i\frac{\pi}{2}} & 0e^{i0} \\ 1e^{i0} & 0.2e^{i\pi} & 0.1e^{i\frac{\pi}{4}} \\ 0e^{i0} & 0.3e^{i\frac{\pi}{3}} & 1e^{i0} \end{bmatrix}_{3\times 3}$ (3)

- **Relative Values**

We can say that, the relative values depend on other values. Here we use the following equation to find the relative values from the known values.

$r_{ij} = f\left(\frac{P_i}{d_j}\right); i = 1, 2, 3, \dots; j = 1, 2, 3, \dots$ (4)

- **Comparison Matrix**

A comparison matrix helps to compare attributes and characteristics of items and helps us to conclude the comparative and relative study. Here we use the following method to create the comparison matrix using the relative values.

Comparison matrix, $R = [r_{ij}];\ r_{ij} = f\left(\frac{P_i}{d_j}\right); i = 1, 2, 3, \dots; j = 1, 2, 3, \dots$ (5)

USE OF COMPLEX FUZZY MATRICES IN THE QUALITY ANALYSIS OF DRINKING WATER

Water has always been an essential and important drink to human being and is necessary for the existence of all organisms. Nowadays, one of the most important issues that human being faces is, the availability of fresh and potable water for human utilization. Among Worldwide, two significant issues in public policies are environmental protection and water quality management. Water composes 70 % of human body by mass, excluding fat.

Over large part of World, human has inadequate access to potable water and drawn water from poisoned lakes, streams and water taps, also use sources contaminated with disease vectors, pathogens or unacceptable level of toxins or suspended solids. According to WHO, almost 80 % of diseases is coming from water and spread through water.

The drinking water quality of any specific area or source can be analyzed using parameters typically fall under two categories (Nidhi *et al*., 2014; Jha *et al.*, 2014).: -

Chemical/ Physical and microbiological. The values of these parameters are unsafe for human health, if they occur more than the defined limits.

The analysis has a lot of confounding alternatives or several parameters of water quality. Therefore, the analysis procedure is based on the study of combinations of variables with complex outcomes. At first, we consider the input variable with their membership functions. In second step, we use the concept of Complex fuzzy set and Complex fuzzy matrices to identify the major parameters that makes the water from the specific source unsafe or unusable.

The input variables are: -

1. pH Factor-d_1
2. Turbidity-d_2
3. Dissolved Oxygen-d_3
4. Biochemical Oxygen Dissolved-d_4
5. Fecal Coliform-d_5

While considering these input variables, we assign some range for defining membership values for these input variables.

pH Factor- d_1

pH is the measure of the intensity of acidity or alkalinity. Acidic water can cause corrosion in pipes and may cause toxic metal from the plumbing system to dissolve in drinking water.

When pH factor rangesbetween 0-6, means that the water is acidic. Then, we use the phase value 0 to represent the range in the complex fuzzy set.

When pH factor range is 6-8.5, means that the water is desirable.We use the phase value $\frac{2\pi}{3}$to represent the range in the complex fuzzy set.

When pH factor range is 8.5-10, means that the water is basic. We use the phase value $\frac{4\pi}{3}$to represent the range in the complex fuzzy set.

The defined membership functions are,

$$\mu_{d_1}(x) = \begin{cases} 0 & ; 0 \leq x \leq 6 \\ \frac{8.5-x}{2.5} & ; 6 < x < 8.5 \\ 0 & ; 8.5 < x \leq 10 \end{cases} \quad (6)$$

$$= \begin{cases} 0e^{i0} \\ \frac{8.5-x}{2.5} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (7)$$

Turbidity- d_2.

Turbidity stands for the measure of relative clarity of water. It is an optical property of a water sample containing insoluble substances which cause light to be scattered rather than transmitted in straight line. Thus, turbidity of a water sample is measured from the amount of light scattered by the water sample in comparison with the standard turbidity. That is, higher the intensity of scattered light, higher the turbidity. High turbidity valued to indicate low water quality. Turbidity can be measured using turbidity meters.

When $d_2$range 0 NTU $\leq$ x $\leq$ 5 NTU (Nephelometer Turbidity Unit) means that, water has low turbidity. Hence,the water is in good quality. we use the phase value 0 to represent the range.

When d_2 range is 5 NTU -7 NTU, the turbidity is medium. Hence, the water is in medium quality. We use the phase value $\frac{2\pi}{3}$ to represent the particular range. \\

When d_2 range 7 NTU -10 NTU, the turbidity is high, so, the water is in poor quality. We use the phase value $\frac{4\pi}{3}$ to represent the particular range.

The defined membership functions are,

$$\mu_{d_2}(x) = \begin{cases} 0 & ; 0 \leq x \leq 5 \\ \frac{7-x}{2} & ; 5 < x < 7 \\ 1 & ; 7 \leq x \leq 10 \end{cases} \quad (8)$$

$$= \begin{cases} 0e^{i0} \\ \frac{7-x}{2} e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \end{cases} \quad (9)$$

Dissolved Oxygen- d_3

Dissolved Oxygen represents the amount of oxygen that is present in the water. A high dissolved oxygen level makes the drinking water tastier. But for industrial water supply, they use water with least possible dissolved oxygen level. Sine high dissolved oxygen level speed up the process of corrosion in water pipes.

Dissolved oxygen level can be measured by three different ways: - titration method, diaphragm electrode method and fluorescence method. The widely used method is diaphragm electrode method.

The variable stands for the dissolved oxygen which ranges from 0 mg/L to 10 mg/L.

When the $d_3$range is $0 \leq d_3 < 4$ dissolved oxygen level is low, we use the phase value 0 to represent the range.
When d_3 range is 4 mg/L - 6.5 mg/L, the dissolved oxygen is in its moderate level, we use the phase value$\frac{2\pi}{4} = \frac{\pi}{2}$ to represent the particular range.
When $d_3$range is6.5 mg/L$\leq d_3 \leq$ 8 mg/L, dissolved oxygen level is high, we use the phase value $\frac{4\pi}{2} = \pi$to represent the particular range.
When d_3 range is 8 mg/L $< d_3 <$ 10, dissolved oxygen level is very high, we use the phase value $\frac{6\pi}{4} = \frac{3\pi}{2}$to represent the particular range.

The defined membership functions are

$$\mu_{d_3}(x) = \begin{cases} 0 & ; 0 \leq x < 4 \\ \frac{6.5-x}{2.5} & ; 4 \leq x < 6.5 \\ \frac{8-x}{1.5} & ; 6.5 \leq x < 8 \\ 1 & ; 8 < x \leq 10 \end{cases} \quad (10)$$

$$= \begin{cases} 0e^{i0} \\ \frac{6.5-x}{2.5} e^{i\frac{\pi}{2}} \\ \frac{8-x}{1.5} e^{i\pi} \\ 1e^{i\frac{3\pi}{2}} \end{cases} \quad (11)$$

Total Hardness-d_4

Hardness of drinking water is defined as the total amount of multivariable metallic cation present in the water. Total hardness is the sum of calcium and magnesium concentration, both of them are expressed as calcium carbonate in milligrams per liter(mg/L).
Total Hardness can be estimated titrimetrically.
The d_4 variable stands for the Total Hardness which ranges from 0 mg/L to 600 mg/L.When the $d_4$range is 0 mg/L $\leq d_4 <$ 75 mg/L, the drinking water is generally considered soft. We use the phase value 0 to represent the range.
When $d_4$range is 76 mg/L - 150 mg/L, the total hardness is in its moderate level, we use the phase value $\frac{2\pi}{4} = \frac{\pi}{2}$to represent the particular range.
When d_4 range is 151 mg/L $\leq d_4 \leq$ 300 mg/L, total hardness level is high, we use the phase value $\frac{4\pi}{4} = \pi$ to represent the particular range.
When d_4 range is 300 mg/L $< d_4 \leq$ 600, total hardness level is very high, we use the phase value $\frac{6\pi}{4} = \frac{3\pi}{2}$to represent the particular range.
The defined membership functions are

$$\mu_{d_3}(x) = \begin{cases} 0 & ; 0 \leq x \leq 75 \\ \frac{150-x}{74} & ; 76 \leq x \leq 150 \\ \frac{300-x}{149} & ; 151 \leq x \leq 300 \\ 1 & ; 300 < x \leq 600 \end{cases} \quad (12)$$

$$= \begin{cases} 0e^{i0} \\ \frac{150-x}{74} e^{i\frac{\pi}{2}} \\ \frac{300-x}{149} e^{i\pi} \\ 1e^{i\frac{3\pi}{2}} \end{cases} \quad (13)$$

Fecal Coliform- d_5

The presence of fecal coliform in a drinking water sample indicates the recent fecal contamination.
The presence of fecal coliform can be analyzed by the method of membrane filtration.
The d_5 variable stands for the Fecal Coliform which ranges from 0 to 5000 and 5000 above.
When the $d_5$range is $0 \leq d_5 < 5000$ fecal coliform level is good. We use the phase value 0 to represent the range.
When d_5 range is > 5000, the fecal coliform level is not good, we use the phase value π to represent the particular range.
The defined membership functions are,

$$\mu_{d_5}(x) = \begin{cases} 0 & ; 0 \leq x < 5000 \\ 1 & ; x \geq 5000 \end{cases} \quad (14)$$

$$= \begin{cases} 0e^{i0} \\ 1e^{i\pi} \end{cases} \quad (15)$$

Remark

In all the membership functions of each fuzzy sets mentioned above, we introduced the phase value to represent the position of that particular input variable in that fuzzy set along with its degree of membership. So that all the above defined fuzzy sets become complex fuzzy sets. This aspect helps us to use the concept of complex fuzzy matrices and which is very helpful in the further developments of decision support system.

Suppose that for a drinking water sample say S_1 pH factor d_1 is 6.25, then the membership value become $\mu_{d_1} = 0.375e^{i\frac{2\pi}{3}}$, obtained by substituting the value 6.25 in the corresponding membership function.

Turbidity= $5 \Longrightarrow \mu_{d_2} = 0e^{i0}$.

Dissolved Oxygen= $5\ mg/L \Longrightarrow \mu_{d_3} = 0.375e^{i\frac{\pi}{2}}$

Total Hardness = $185mg/L \Longrightarrow \mu_{d_4} = 0.762e^{i\pi}$

Fecal Coliform= $4000 \Longrightarrow \mu_{d_5} = 0e^{i0}$

The first-row matrix for the SampleS_1 is,

$$S_1 \rightarrow \begin{bmatrix} 0.375e^{i\frac{2\pi}{3}} \\ 0e^{i0} \\ 0.375e^{i\frac{\pi}{2}} \\ 0.762e^{i\pi} \\ 0e^{i0} \end{bmatrix}^T \quad (16)$$

The second-row matrix for the sampleS_2 is,

$$S_2 \rightarrow \begin{bmatrix} 0.142e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 0.154e^{i\pi} \\ 1e^{i\frac{3\pi}{2}} \\ 1e^{i\pi} \end{bmatrix}^T \quad (17)$$

The third-row matrix for the sampleS_3 is,

$$S_3 \rightarrow \begin{bmatrix} 0.213e^{i\frac{2\pi}{3}} \\ 1e^{i\frac{4\pi}{3}} \\ 0.625e^{i\frac{\pi}{2}} \\ 0.657e^{i\frac{\pi}{2}} \\ 1e^{i\pi} \end{bmatrix}^T \quad (18)$$

The fourth-row matrix for the sampleS_4 is,

$$S_4 \rightarrow \begin{bmatrix} 1e^{i\frac{2\pi}{3}} \\ 0e^{i0} \\ 0.125e^{i\frac{\pi}{2}} \\ 0.589e^{i\pi} \\ 0e^{i0} \end{bmatrix}^T \quad (19)$$

The fifth-row matrix for the sampleS_5 is,

$$S_5 \rightarrow \begin{bmatrix} 0.3667e^{i\frac{2\pi}{3}} \\ 0.5e^{i\frac{2\pi}{3}} \\ 0.1538e^{i\pi} \\ 0.0789e^{i\frac{\pi}{2}} \\ 1e^{i\pi} \end{bmatrix}^T \quad (20)$$

So, the Age group- Cause complex fuzzy matrix is,

$$\begin{bmatrix} 0.375e^{i\frac{2\pi}{3}} & 0e^{i0} & 0.375e^{i\frac{\pi}{2}} & 0.762e^{i\pi} & 0e^{i0} \\ 0.142e^{i\frac{2\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 0.154e^{i\pi} & 1e^{i\frac{3\pi}{2}} & 1e^{i\pi} \\ 0.213e^{i\frac{2\pi}{3}} & 1e^{i\frac{4\pi}{3}} & 0.625e^{i\frac{\pi}{2}} & 0.657e^{i\frac{\pi}{2}} & 1e^{i\pi} \\ 1e^{i\frac{2\pi}{3}} & 0e^{i0} & 0.125e^{i\frac{\pi}{2}} & 0.589e^{i\pi} & 0e^{i0} \\ 0.3667e^{i\frac{2\pi}{3}} & 0.5e^{i\frac{2\pi}{3}} & 0.1538e^{i\pi} & 0.0789e^{i\frac{\pi}{2}} & 1e^{i\pi} \end{bmatrix}_{5\times 5} \quad (21)$$

Calculate the relative values $r_{ij} = f\left(\frac{S_i}{d_j}\right)$ and form the comparison matrix R.

$$R = [r_{ij}]_{5\times 5} = \left[f\left(\frac{S_i}{d_j}\right)\right]_{5\times 5} ; i,j = 1,2,3,4,5 \quad (22)$$

$$r_{11} = f\left(\frac{S_1}{d_1}\right) = \frac{|\mu_{d_1}(S_1)| - |\mu_{S_1}(d_1)|}{max\{|\mu_{d_1}(P_1)|, |\mu_{P_1}(d_1)|\}} = \frac{0.375-0.375}{max\{0.375,0.375\}} = \frac{0}{0.375} = 0 \quad (23)$$

$$r_{12} = f\left(\frac{S_1}{d_2}\right) = \frac{|\mu_{d_2}(S_1)| - |\mu_{S_1}(d_2)|}{max\{|\mu_{d_2}(S_1)|, |\mu_{S_1}(d_2)|\}} = -1 \quad (24)$$

$$r_{13} = f\left(\frac{S_1}{d_3}\right) = \frac{|\mu_{d_3}(S_1)| - |\mu_{S_1}(d_3)|}{max\{|\mu_{d_3}(S_1)|, |\mu_{S_1}(d_3)|\}} = 0.432$$

$$r_{14} = f\left(\frac{S_1}{d_4}\right) = \frac{|\mu_{d_4}(S_1)| - |\mu_{S_1}(d_4)|}{max\{|\mu_{d_4}(S_1)|, |\mu_{S_1}(d_4)|\}} = -0.238$$

$r_{15} = f\left(\frac{S_1}{d_5}\right) = -1$; $r_{21} = f\left(\frac{S_2}{d_1}\right) = 1$; $r_{22} = f\left(\frac{S_2}{d_2}\right) = 0$; $r_{23} = f\left(\frac{S_2}{d_3}\right) - 0.846$; $r_{24} = f\left(\frac{S_2}{d_4}\right) = 1$;
$r_{25} = f\left(\frac{S_2}{d_5}\right) = 0.5$

Similarly, we get all the remaining relative values,

$r_{31} = -0.432;\ r_{32} = 0.846;\ r_{33} = 0;\ r_{34} = 1;\ r_{35} = 0.8462$
$r_{41} = 238;\ r_{42} = -1;\ r_{43} = -1;\ r_{44} = 0;\ r_{45} = -1$
$r_{51} = 1;\ r_{52} = -0.5;\ r_{53} = -0.8462;\ r_{54} = 1;\ r_{55} = 0$

So, the comparison matrix is,

$$\begin{bmatrix} 0.0000 & -1.000 & 0.43200 & -0.238 & -1.000 \\ 1.0000 & 0.0000 & -0.8460 & 1.0000 & 0.5000 \\ -0.438 & 0.8460 & 0.00000 & 1.0000 & 0.8462 \\ 0.2380 & -1.000 & -1.0000 & 0.0000 & -1.000 \\ 1.0000 & -0.500 & -0.8462 & 1.0000 & 0.0000 \end{bmatrix}_{5\times 5} \quad (25)$$

From the ranking of the problem, we can conclude that, the main cause of unsafe nature of factors of drinking water in each sample can be found by taking maximum in each row, so for sample S_1 isd_3 i.e. the dissolved oxygen, for sample S_2 the content d_1 and d_4 i.e. the pH factor and Total hardness, for sample S_3 the content d_4 i.e. the contents Total hardness, for sample S_4 the content d_1 i.e. the content pH factor, for the sample S_5 the contents d_1 and d_4 i.e. the contents pH factor and Total hardness.

CONCLUSION

In this paper we use the concept of complex fuzzy matrices. The concept of complex fuzzy sets has undergone an evolutionary process since they first introduced. Medical fields and decision-making algorithms are the major field in which the concept of complex fuzzy sets and complex fuzzy matrices are applicable. Use of the theory of fuzzy matrices in the field of human diseases diagnosis and decision-making algorithms was recognized quite early. The decision maker generally gathers knowledge about the samples of the test results and various dependent factors, the knowledge provided by each of these factors carries with it varying degrees of uncertainty. This problem can be overcome by using the concept of fuzzy matrices.

Hence in this paper, by the utilization of the new introduced concept of complex fuzzy matrices, we identify the key factors that make various drinking water samples unsafe. This method can assist the analyst in identifying the major causes, and helps him to find some methods of making the sample safe to drink. In future this method can be applied for different types of analysis and in the decision making for different disease management.

REFERENCES

1. Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control, Netherlands,8,* 338-353 (Zadeh, 1965).
2. Thomson, M.G. (1997). Convergence of Powers of Fuzzy Matrix. *Math. Anal. Appl., 57*(2), 476-480 (Thomson, 1997).
3. Ramot, D., Milo, R., Friedman, M. and Kandel, A. (2002). Complex Fuzzy Sets. *IEEE Transactions on Fuzzy Systems, 10*(2), 171-186 (Ramot*et al*., 2002; Milo *et al*., 2002; Friedman *et al*., 2002; Kandel *et al.*, 2002)
4. Tamir, D. E., Jin, L. and Kandel, A. (2011).A new Interpretation of Complex Membership Grade. *International Journal of Intelligent System, New Jersey, 26*(4), 285-312 (Tamir *et al*., 2011; Jin*et al*., 2002; Kandel *et al*., 2011).
5. Nidhi Mishra and Jha, P. (2014). Fuzzy Expert System for Drinking Water Quality Index. *Recent Research in Science and Technology, 6*(1), 122-125(Nidhi *et al*., 2014; Jha *et al*., 2014).
6. Geetha, T. and Usha, A. (2017). Circulant Fuzzy Matrices in Human Body Diseases. *International Journal of Pure and Applied Mathematics, 117*(6), 99-106 (Geetha *et al*., 2017; Usha *et al.*, 2017).
7. ThankarajBeaula and Mallika (2017). Applications of Fuzzy Matrices in Medical Diagnosis. *International Journal of Fuzzy Mathematical Achieve,14*(1), 163-169 (Beaula*et al*., 2017; Millika*et al*., 2017).