BioGecko

Vol 12 Issue 03 2023
ISSN NO: 2230-5807

SOME RESULTS ON GROUP S_3 CORDIAL SUM DIVISOR LABELING.

M.Seenivasan, P.Aruna Rukmani and A.Lourdusamy

M.Seenivasan,
Department of Mathematics,
Associate Professor,
Sri Paramakalyani College,
Alwarkurichi-627412,
Tamilnadu,India.
E-mail address: msvasan_22@yahoo.com

P.Aruna Rukmani,
Department of Mathematics,
Research scholar, Registration number:19121282092009.
Affliated to Manonmaniam Sundaranar University,
Abishekapatti.,Tirunelveli-627012,
Tamilnadu,India.
E-mail address: parukmanimaths@gmail.com

A.Lourdusamy,
Department of Mathematics,
Associate Professor,
St.Xavier's college(Autonomous),
Palayamkottai-627002,
Tamilnadu, India.
E-mail address: lourdusamy15@gmail.com

Abstract
Let $G = (V(G), E(G))$ be a graph and let $h: V(G) \to S_3$ be a function.For each edge uv assign the label 1 if 2 divides $(O(h(u)) + O(h(v)))$ and 0 otherwise. The function h is called a Group S_3 cordial sum divisor labeling of G if $|v_h(i) - v_h(j)| \leq 1$ and $|e_h(1) - e_h(0)| \leq 1$ where $v_h(k)$ denote the number of vertices labeled with k, $k \in S_3$ and $e_h(1)$ and $e_h(0)$ denote the number of edges labeled with 1 and 0 respectively. A graph G which admits a Group S_3 cordial sum divisor labeling is called a Group S_3 cordial sum divisor graph. We investigate some results on Group S_3 cordial sum divisor labeling.

Keywords: Sum divisor cordial labeling, Group S_3 cordial sum divisor labelling

1. INTRODUCTION

A Graph labeling is an assignment of integers to the vertices or edges,or both,subject to certain conditions. Graph labelings were first introduced in the mid 1960's.For graph theoretical terminology, we refer to [2]. All graphs considered here are simple,finite and undirected. For a detailed survey of graph labeling refer [3]. We denote the vertex set and edge set of G by $V(G)$ and $E(G)$ so that the order and size of G are $|V(G)|$ and $|E(G)|$ respectively. Cordial labeling is a weaker version of graceful labeling and Harmonious labeling introduced by I.Cahit in [1].

The concept of Sum divisor cordial labeling was due to Lourdusamy and Patrick [5]. The concept of Group S_3 cordial prime labeling was due to Kala and Chandra [4]. The concept of Group S_3 cordial sum divisor labeling was due to M.Seenivasan,P.Aruna Rukmani and A.Lourdusamy [6]. In this paper we investigate some results on Group S_3 cordial sum divisor labeling.

Definition 1.1 Let A be a group. The order of an element $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$.

Definition 1.2

Consider the Symmetric group S_3=$\{e, a, b, c, d, f\}$ where

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$
$$a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$
$$b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$
$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$
$$d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$
$$f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

we have $O(e) = 1, O(a) = O(b) = O(c) = 2$,and $O(d) = O(f) = 3$.

2. MAIN RESULTS

Definition 2.1. Let $G = (V(G), E(G))$ be a graph and let $h: V(G) \rightarrow S_3$ be a function. For each edge uv assign the label 1 if 2 divides $(O(h(u)) + O(h(v)))$ and 0 otherwise. The function h is called a Group S_3 cordial sum divisor labeling of G if $|v_h(i) - v_h(j)| \leq 1$ and $|e_h(1) - e_h(0)| \leq 1$,where $v_h(k)$ denote the number of vertices labeled with $k, k \in S_3$ and $e_h(1)$ *and* $e_h(0)$ denote the number of edges labeled with 1 and 0 respectively. A graph G which admits a Group S_3 cordial sum divisor labeling iscalled a Group S_3 cordial sum divisor graph.

Definition 2.2. $H(2n, 2n + 1), (n = 2,3,4, \dots), (t = 0,1,2,3, \dots)$ denote the graph obtained from a Cycle C_{2n} by adding $2t + 1$ consecutive diagonals including the central diagonal and $2t$ diagonals symmetric to the central diagonal. If $n = m$, a positive integer then t will take only the values $0,1,2, \dots,, m - 2$.

Theorem 2.1 $H(2n, 2n + 1)$ is a Group S_3 cordial sum divisor graph for $n = 2,3,4,5,6$.

Proof. Let $G = H(2n, 2n + 1)$ be a graph with 'p' vertices and 'q' edges.

Case 1. $n = 2, t = 0$.

The graph $G = H(4,1)$ has 4 vertices and 5 edges.

Let v_1, v_2, v_3, v_4 be the vertices and the edge set is defined as

$\{(v_i v_{i+1}): 1 \leq i \leq 3 \cup (v_4, v_1), (v_1, v_3)\}$

Define $h: V(G) \rightarrow S_3$ as follows:

$h(v_1) = a$
$h(v_2) = b$
$h(v_3) = d$
$h(v_4) = c$

It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.

Therefore, h is a group S_3 cordial sum divisor labeling.

Hence, the graph H(4,1) is a Group S_3 cordial sum divisor graph.

Case 2. $n = 3, t = 0,1$.

Table 1

n	t	$H(2n, 2t + 1)$	p	q
3	0	$H(6,1)$	6	7
3	1	$H(6,3)$	6	9

Now, For the graphs H(6,1) and H(6,3). Define $h: V(G) \rightarrow S_3$ as follows:
$h(v_1) = a$
$h(v_2) = b$
$h(v_3) = e$
$h(v_4) = d$
$h(v_5) = f$
$h(v_6) = c$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, the graphs H(6,1) and H(6,3) is a Group S_3 cordial sum divisor graph.
Case 3. $n = 4, t = 0,1,2$.

Table 2

n	t	$H(2n, 2t + 1)$	p	q
4	0	$H(8,1)$	8	9
4	1	$H(8,3)$	8	11
4	2	$H(8,5)$	8	13

Now, For the graphs H(8,1) and H(8,3). Define $h: V(G) \rightarrow S_3$ as follows:
$h(v_1) = a$
$h(v_2) = b$
$h(v_3) = e$
$h(v_4) = c$
$h(v_5) = d$
$h(v_6) = f$
$h(v_7) = e$
$h(v_8) = c$
Also, For the graph H(8,5). Define $h: V(G) \rightarrow S_3$ as follows:
$h(v_1) = a$
$h(v_2) = b$
$h(v_3) = e$
$h(v_4) = c$
$h(v_5) = d$
$h(v_6) = f$
$h(v_7) = a$
$h(v_8) = c$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.

Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, the graphs H(8,1), H(8,3) and H(8,5) is a Group S_3 cordial sum divisor graphs.
Case 4. $n = 5, t = 0,1,2,3.$

Table 3

n	t	$H(2n, 2t + 1)$	p	q
5	0	$H(10,1)$	10	11
5	1	$H(10,3)$	10	13
5	2	H(10,5)	10	15
5	3	H(10,7)	10	17

Now, For the graphs H(10,1),H(10,3),H(10,5) and H(10,7).
Define $h: V(G) \rightarrow S_3$ as follows:
$h(v_1) = a$
$h(v_2) = b$
$h(v_3) = a$
$h(v_4) = c$
$h(v_5) = e$
$h(v_6) = d$
$h(v_7) = f$
$h(v_8) = d$
$h(v_9) = c$
$h(v_{10}) = e$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, the graphs H(10,1), H(10,3),H(8,5) and H(10,7) is a Group S_3 cordial sum divisor graphs.
Case 5. $n = 6, t = 0,1,2,3,4.$

Table 4

n	t	$H(2n, 2t + 1)$	p	q
6	0	$H(12,1)$	12	13
6	1	$H(12,3)$	12	15
6	2	H(12,5)	12	17
6	3	H(12,7)	12	19
6	4	H(12,9)	12	21

Now, For the graphs H(12,1),H(12,3),H(12,5),H(12,7) and H(12,9).
Define $h: V(G) \rightarrow S_3$ as follows:
$h(v_1) = h(v_3) = a$
$h(v_2) = h(v_9) = b$
$h(v_4) = h(v_{11}) = c$

$h(v_5) = h(v_8) = f$
$h(v_6) = h(v_{12}) = e$
$h(v_7) = h(v_{10}) = d$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, the graphs H(12,1), H(12,3),H(12,5),H(12,7) and H(12,9) is a Group S_3 cordial sum divisor graphs.

Table 5

n	t	$H(2n, 2t+1)$	$e_g(0)$	$e_g(1)$
2	0	H(4,1)	3	2
3	0	H(6,1)	3	4
3	1	H(6,3)	5	4
4	0	H(8,1)	5	4
4	1	H(8,3)	6	5
4	2	H(8,5)	7	6
5	0	H(10,1)	5	6
5	1	H(10,3)	7	6
5	2	H(10,5)	8	7
5	3	H(10,7)	8	9
6	0	H(12,1)	7	6
6	1	H(12,3)	8	7
6	2	H(12,5)	9	8
6	3	H(12,7)	10	9
6	4	H(12,9)	10	11

Definition 2.3 The Durer graph is an undirected graph with 12 vertices and 18 edges,formed by the connection vertex to vertex between a regular hexagon and a six point star..
Theorem 2.2 The Durer graph is not a group S_3 cordial sum divisor graph.
Theorem 2.3 The graph obtained by the path union of two copies of Durer graph is a group S_3 cordial sum divisor graph.
Proof.
Let G_1 and G_2 be the two copies of Durer graphs.
Let $V(G_1) = \{u_i; 1 \leq i \leq 12\}$ and $V(G_2) = \{v_i; 1 \leq i \leq 12\}$.
$E(G_1) = \{[(u_1u_{12}), (u_iu_{i+1}): 1 \leq i \leq 11] \cup [(u_1u_9), (u_2u_7), (u_3u_5), (u_4u_{12})(u_6u_{11}), (u_8u_{10})]\}$

$E(G_2) = \{[(v_1v_{12}), (v_iv_{i+1}): 1 \leq i \leq 11] \cup [(v_1v_9), (v_2v_7), (v_3v_5), (v_4v_{12}), (v_6v_{11}), (v_8v_{10})]\}$
Let G be the graph obtained by the path union of copies of Durer graphs G_1 and G_2.
Here,we add a path of length 3 from G_1 to G_2.
Then $V(G) = V(G_1) \cup V(G_2) \cup V(w_1) \cup V(w_2)$ and $E(G) = E(G_1) \cup E(G_2) \cup E(u_{11}w_1) \cup E(w_1w_2) \cup E(w_2v_9)$.
Here, G has 26 vertices and 39 edges.
Now,we define $h: V(G) \to S_3$ as follows:
$h(u_1) = h(u_9) = h(v_1) = h(v_9) = e$
$h(u_2) = h(u_{11}) = h(v_2) = h(v_{11}) = f$
$h(u_3) = h(u_6) = h(v_3) = h(v_6) = c$
$h(u_4) = h(u_{10}) = h(v_4) = h(v_{10}) = b$
$h(u_5) = h(u_8) = h(v_5) = h(v_8) = a$
$h(u_7) = h(u_{12}) = h(v_7) = h(v_{12}) = d$
$h(w_1) = d$
$h(w_2) = b$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i,j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence the graph obtained by the path union of two copies of Durer graph is a group S_3 cordial sum divisor graph.

Table 6

Vertex and Edge	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$	$e_h(0)$	$e_h(1)$
$Number$	4	5	4	5	4	4	19	20

Definition 2.4 The Truncated tetrahedron graph is formed with 12 vertices and 18 edges.It is a connected cubic transitive graph.
Theorem 2.4 A Truncated tetrahedron graph is a group S_3 cordial sum divisor graph.
Proof. Let $G = (V, E)$ be a truncated tetrahedron graph with 12 vertices and 18 edges. Let $V(G) = \{u_i; 1 \leq i \leq 12\}$ and
$E(G) = \{[(u_1u_8), (u_iu_{i+1}): 1 \leq i \leq 7] \cup [(u_9u_1), (u_9u_8), (u_9u_{11})] \cup [(u_{10}u_2), (u_{10}u_3), (u_{10}u_{12})] \cup [(u_{11}u_4), (u_{11}u_5)] \cup [(u_{12}u_6), (u_{12}u_7)]\}$.
Now,we define $h: V(G) \to S_3$ as follows:
$h(u_1) = h(u_4) = b$
$h(u_2) = h(u_{12}) = d$
$h(u_3) = h(u_7) = f$
$h(u_5) = h(u_8) = c$
$h(u_9) = h(u_{10}) = a$
$h(u_6) = h(u_{11}) = e$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i,j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, Truncated tetrahedron graph is a group S_3 cordial sum divisor graph.

Definition 2.5 The Frucht graph is formed with 12 vertices and 18 edges.It is a 3-regular graph and it has no non-trivial symmetries.
Theorem 2.5 The Frucht graph is a group S_3 cordial sum divisor graph.
Proof. Let $G = (V, E)$ be a Frucht graph with 12 vertices and 18 edges.
Let $V(G) = \{u_i; 1 \leq i \leq 12\}$ and

$$E(G) = \{[(u_1u_7),(u_iu_{i+1}): 1 \le i \le 6] \cup [(u_1u_8),(u_2u_8),(u_8u_{11})] \cup [(u_3u_9),(u_4u_9),(u_9u_{12})] \cup [(u_5u_{10}),(u_6u_{10}),(u_{10}u_{12})] \cup [(u_7u_{11}),(u_{11}u_{12})].$$

Now,we define $h: V(G) \to S_3$ as follows:
$h(u_1) = h(u_6) = a$
$h(u_2) = h(u_{12}) = f$
$h(u_3) = h(u_{11}) = d$
$h(u_4) = h(u_{10}) = c$
$h(u_5) = h(u_8) = b$
$h(u_7) = h(u_9) = e$
It is easy to verify that $|v_h(i) - v_h(j)| \le 1$ for every $i,j \in S_3$ and $|e_h(1) - e_h(0)| \le 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, Frucht graph is a group S_3 cordial sum divisor graph.

Theorem 2.6 The Tieze graph is not a group S_3 cordial sum divisor graph.
Theorem 2.7 The graph obtained by the path union of two copies of Tieze graph is a group S_3 cordial sum divisor graph.
Definition 2.6 The Wagner graph is formed with 8 vertices and 12 edges. All the vertices of this graph can be arranged in a cycle and each vertex is joined with the other vertices whose portion is differ by a number 1(modulus 3).
Theorem 2.8 The Wagner graph is a group S_3 cordial sum divisor graph.
Proof. Let G be the Wagner graph.
Let $V(G) = \{u_i; 1 \le i \le 8\}$ and

$$E(G) = \{[(u_1u_8),(u_iu_{i+1}): 1 \le i \le 7] \cup [(u_iu_{i+4}): 1 \le i \le 4\}]$$

Now,we define $h: V(G) \to S_3$ as follows:
$h(u_1) = h(u_4) = d$
$h(u_2) = e$
$h(u_3) = h(u_7) = a$
$h(u_5) = b$
$h(u_6) = f$
$h(u_8) = c$
It is easy to verify that $|v_h(i) - v_h(j)| \le 1$ for every $i,j \in S_3$ and $|e_h(1) - e_h(0)| \le 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence, Wagner graph is a group S_3 cordial sum divisor graph.

Definition 2.7 Theta graph is a segment with non-adjacent vertices of degree 3 and all the other vertices of degree 2 is called a Theta graph.
Theorem 2.10 The Theta graph T is a Group S_3 cordial sum divisor graph.
Proof. Let $v_0, v_1, v_2, \cdots, v_6$ be the vertices of the Theta graph with centre v_0 and the edge set $E(T) = \{v_iv_{i+1}; 1 \le i \le 5\} \cup \{v_0v_1, v_0v_4, v_1v_6\}$
Here,|V(T)|=7 and |E(T)|=8
Now,Define $h: V(T) \to S_3$ is as follows
$h(v_1) = e$
$h(v_2) = a$
$h(v_3) = b$
$h(v_4) = d$
$h(v_5) = c$
$h(v_0) = h(v_6) = f$
Now, we observe that $e_h(0) = 4, e_h(1) = 4$.

Table 7

Vertices	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$
Number	1	1	1	1	1	2

From TABLE 7,It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, h is a Group S_3 cordial sum divisor labeling.

Definition 2.8 Let G be a graph and let $G_1 = G_2 = \cdots G_n$ where $n \geq 2$, then the graph obtained by adding an edge from each G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called the path union of G.

Theorem 2.10 The graph obtained by path union of two copies of Theta graph T is a Group S_3 cordial sum divisor graph.

Proof. Let $v_0, v_1, v_2, \cdots, v_6$ be the vertices of the Theta graph T^1 with centre v_0 and the edge set $E(T^1) = \{v_i v_{i+1}; 1 \leq i \leq 5\} \cup \{v_0v_1, v_0v_4, v_1v_6\}$

Let $w_0, w_1, w_2, \cdots, w_6$ be the vertices of the Theta graph T^2 with centre w_0 and the edge set $E(T^2) = \{w_i w_{i+1}; 1 \leq i \leq 5\} \cup \{w_0w_1, w_0w_4, w_1w_6\}$

Here,$|V(T^1)| = 7$ and $|E(T^1)| = 8$ and $|V(T^2)| = 7$ and $|E(T^2)| = 8$.

Let G be the graph obtained by the path union of copies of Theta graphs T^1 and T^2.

Here, we add an edge from T^1 to T^2.

Then $V(G) = V(T^1) \cup V(T^2)$ and $E(G) = E(T^1) \cup E(T^2) \cup \{v_3w_2\}$.

Here G has 14 vertices and 17 edges.

Now,Define $h: V(G) \to S_3$ is as follows

$h(v_1) = e, h(v_2) = a, h(v_3) = b$

$h(v_4) = d, h(v_5) = c, h(v_0) = g(v_6) = f$

$h(w_1) = e, h(w_2) = a, h(w_3) = b$

$h(w_4) = h(w_0) = d,$

$h(w_5) = c,$

$h(w_6) = f$

Now, we observe that $e_h(0) = 8, e_h(1) = 9$.

Table 8

Vertices	$v_h(a)$	$v_h(b)$	$v_h(c)$	$v_h(d)$	$v_h(e)$	$v_h(f)$
Number	2	2	2	3	2	3

From TABLE 8,It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$ Therefore, h is a Group S_3 cordial sum divisor labeling.

Hence, the graph obtained by path union of two copies of Theta graph T is a Group S_3 cordial sum divisor graph.

Definition 2.9 A Herschel graph H_s is a bipartite undirected graph with 11 vertices and 18 edges.

Theorem 2.11 The Herschel graph H_s is a Group S_3 cordial sum divisor graph.

Proof. Let $G = H_s$ be a Herschel graph. Let W be the central vertex and $u_i; (1 \leq i \leq 10)$ be the remaining vertices of the Herschel graph.

Then $|V(G)| = 11$ and $|E(G)| = 18$

Now,we define $h: V(G) \to S_3$ as follows:

$h(u_1) = h(u_9) = a$

$h(w) = e$
$h(u_2) = h(u_5) = b$
$h(u_3) = h(u_8) = c$
$h(u_4) = h(u_6) = d$
$h(u_7) = f$
$h(u_{10}) = e$
It is easy to verify that $|v_h(i) - v_h(j)| \leq 1$ for every $i, j \in S_3$ and $|e_h(1) - e_h(0)| \leq 1$.
Therefore, h is a group S_3 cordial sum divisor labeling.
Hence,the Herschel graph H_s graph is a group S_3 cordial sum divisor graph.

REFERENCES

1. I.Cahit, *Cordial graphs:A weaker version of Graceful and Harmonious graphs*,Ars Combin.,vol.23(1987).
2. F. Harary, *Graph Theory*, Addison-wesley, Reading, Mass 1972.
3. J. A. Gallian, *A Dyamic Survey of Graph Labeling*, The Electronic J. Combin., 21 (2020) # DS6.
4. Kala and Chandra, *Group S_3 cordial prime labeling of Graphs*,Malaya Journal of Mathematik.,vol.1,(2019).
5. Lourdusamy and Patrick, *Sum Divisor Cordial Graphs*,Proyecciones Jounal of mathematics.,volume 35,(2016).
6. M.Seenivasan, P.Aruna Rukmani and A.Lourdusamy, *Group S_3 Cordial Sum Divisor Labeling Of Some Derived Graphs*, Special Ugdymas-Special education;volume 2, Issue 43, 2022,PP.2445-2453, (UGC approved Journal).
7. M.Seenivasan, P.Aruna Rukmani and A.Lourdusamy,*Group S_3 Cordial Sum Divisor Labeling for Some Graphs* ,International Conference on Recent Developments in Pure and Applied Mathematics" July 19 to 21 2022., Conference Proceedings ICRDPAM 2022-ISBN NUMBER :978-93-5680-006-9, volume 1,pp:128-136,(July 19 to 21 2022).