

## The Relationship Between Airline Ticket Pricing Model and Demand Uncertainty (Case Study: Iran Airtour Airline)

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### Abstract

The present study considered a problem in optimizing the price of airline tickets regarding the demand uncertainty, ticket cancellation rate, absence rate, and airline ticket classification. In addition, the relevant mathematical model was developed by considering the multiplicative uncertainty model. Genetic algorithm and LINGO were used to solve the model since the model was a mixed integer non-linear mathematical programming and its objective function was non-concave. The optimal ticket prices were obtained by considering the limitations raised in the model. Eventually, comparing the results of the genetic algorithm to Lingo showed that the genetic algorithm results were more satisfactory. The observations and results on a sample of 30 random problems revealed that the genetic algorithm developed in this study is appropriate for solving the model with actual data in the real world.

**Key words;** Airline ticket pricing, mathematical modeling, price optimization, genetic algorithm

### Introduction

Pricing as one of the effective factors for the introduction of products has a long history. Price is the only characteristic of a product representing its quality and other characteristics. In other words, price somehow connects the customer's mentality and the activities of product supply (from manufacturing to sales) since the customer often considers the price as a factor for finding the quality or gaining additional information which is highly time-consuming (Simon, 1968). Thus, the significance of price and appropriate pricing is not hidden from anybody. Today, pricing just by considering the supply and demand is impossible due to the nature of the product (such as perishable or high-tech products) and mostly the competitive nature of the market. Airlines should adopt new pricing policies by considering past procedures to maximize their profits. In each period, airlines should provide pricing according to the amount of demand regarding several classes of customers, ticket cancellations, and absences.

Bitran and Caldentri conducted a review study on pricing models in revenue management. In all the models in their review study, the seller used a set of resources to manufacture products for uncertain price-sensitive promotion at a limited time (Biller,2005). Elmaghraby and Keskinocak (2003) conducted a comprehensive review of different types of research about different states of the dynamic pricing problem with inventory considerations. Furthermore, they proposed diverse, practical, and interesting suggestions for future studies in this field. Chiang et al. carried out a review study on existing research topics in revenue management and addressed the pricing problem in part of this review study. In addition, they reviewed some articles related to this problem and finally raised several suggestions for future studies in the field of revenue management (Chao, 2006). Zabel and Tavsen studied pricing and production rate when demand is considered as a function of price or a

possible component as a whole. The costs considered in this study included holding costs, ordering costs, and shortage costs. Zabel evaluated his problem in various states of backlog, partial backlog, and lost sales (Martinez, 1970). Birg considered the optimal pricing strategies for two substitute products of A and B in a single period with capacity limitation. Furthermore, it was assumed that the demand for products has a uniform distribution and the final revenue of the company is achieved from the total revenues from products A and B (Feng, 1995). The presence of a mathematical model to optimize the price of an airline ticket seems obvious due to the government's policies for the liberalization and competitiveness of airline ticket prices, the arrival of new companies into the air transportation market, and diversity in the priorities and tastes of customers for selecting airline companies, economic efficiency and the current limitations of services by airline companies for maximizing profits or reducing costs. Assume an airline with a limited number of airplanes for providing customer services (the number of airplanes is considered in this problem) on a certain route with a resource called seats and two products in classes A and B that the price in one class can affect demand in the next class. The company encounters two choices for selling each ticket and can book the ticket at a lower price (that is one of the variables of the problem and should be determined) at the second class or keep it for late bookings when the full fare is charged with no discount. For each of these two classes, a demand exists which is a function of the price offered for that class, indicating the average ticket demand. The demand for each product has a multiplicative uncertainty, the parameter of which has a continuous uniform distribution. In this model, the demand changes by multiplying a random variable, and naturally, the mathematical expectation of this random variable equals one. The price in the first class should be higher than the price in the second class while the price for both classes is placed within a limited range. The price for each class has an upper limit (maximum price) and a lower limit (lower limit) that is applied as a limitation in the problem. The number of seats is limited and customers can cancel the ticket. Based on the time passed after the ticket purchase, different percentages of the ticket price can be paid back to the customers. Furthermore, the possibility of customer absence on the flight is regarded while its average price is disregarded during the sale (double booking is possible). Accordingly, this study aims to investigate the relationship between the airline ticket pricing model and demand uncertainty.

### **Revenue Management**

Revenue management addresses the decisions based on demand management and its required methods and systems. This involves the common management of the company and the market to increase revenue. Revenue management can be regarded as a supplement to the supply chain for the supply processes and decisions in an enterprise to reduce production and delivery cost (Jeffrey, 1999).

In addition, revenue management or yield management can be defined as follows: "The art of maximizing the profits obtained from the sale of a limited capacity of a perishable product at a certain time by selling each unit of product to the right customer at the right time and a reasonable price. Boyd and Bilgan reviewed more than 110 articles on revenue management and could present a successful model for e-commerce reservation, sales, and revenue management systems (Jobber,1998). Furthermore, Urban and Baker considered certain demand as a function of several variables such as price, time, and inventory level. In addition, they considered downward price policies and provided the approach to determining the optimal order quantity policy (Urban and Baker, 1997). Waterford studied the problem of allocation and pricing simultaneously for various price classes which are used in the transportation industry. In addition, demand was assumed to be normally distributed for multiple classes. In this study, the limitations were considered as the upper limit for the values of the class allocation. Moreover, the researcher aimed to maximize the profits according to price ranking and other limitations. Different behaviors of customers and control mechanisms were evaluated in his

study (Taylor, 1962).

### **Pricing principles**

Nowadays, pricing is considered a critical tool in maximizing the profits of companies. This section deals with the existing pricing models, their theoretical characteristics, and some of their uses in various industries. Companies select the optimal price from a set of predetermined prices or assume the demand to be certain for the product. Due to the emergence of new companies in the competitive market and the lack of harmony among customers' tastes, considering product differences, different demands, and a wider range of prices seems inevitable in pricing models. For a more comprehensive review, the present study classified different problems in terms of the considered assumptions, the number of parameters, and the characteristics of the intended problem. Considering some assumptions such as the certainty or probability of the parameters needs different tools and parameters for solving the problem. Here are some of the most common assumptions for each parameter and the literature review.

*Length of time horizon:* To investigate this parameter, many articles have considered time as a single period. However, some articles considered the time factor as two or more periods.

*Price:* While facing the pricing policy, the problems which consider the time horizon as several periods are divided into two groups. In the first group such as traditional business models, the price remains constant during the time horizon (even if the demand is not constant during the planning period). In the second group, the price dynamically changes over time (the price is assumed to be dynamic and a function of parameters such as demand, inventory, etc. in such problems).

*Demand type:* Demand can be deterministic or probabilistic. Deterministic demand refers to a well-known function of parameters such as price. The economic order model is an example of deterministic demand. It is assumed that probabilistic demand is a function of price which is summed with a probabilistic component.

*Form of demand function:* In the pricing literature, there are different types of relationships between demand and price or other parameters such as inventory. In addition, demand is seen as a linear function of price in its most common form.  $p + a = D$  and if the demand is probabilistic, the probabilistic component will be added to it.  $p + a = D$  or  $p + a = D$ . In addition, the demand is common in literature with exponential or Poisson distribution where it is assumed that the arrival of customers occurs in independent time units.

*Demand input parameters:* Demand is considered a function of price in the most general form. Nevertheless, there are numerous parameters that may affect the demand. Such parameters include time, inventory, promotions, advertisements, and product features.

*Sales:* Researchers have provided some hypotheses for facing excess demand when the demand is probabilistic or there is a capacity limitation in the problem. Some studies assumed that the excess demand faces backlog and is answered in the next period and sometimes the excess demand is considered as lost sales. None of the above-mentioned assumptions are required when the demand is deterministic or the price is determined in a way to meet the demand precisely.

*Capacity limitation:* Production problems are classified into two groups in terms of capacity limitation. In one classification, production may be limited by system capacity while no capacity is considered in the other classification. In addition, the literature review indicates that most articles avoided considering the capacity limitation. Products: In the pricing literature, products are regarded as single or multiple with no sharing of resources. Few studies have considered the pricing of multiple products in line with resource sharing (Smith et al, 1998).

**Method**

The present study was descriptive-mathematical and the required data were obtained through the study. Data analysis was of mathematical modeling type and quantitative analysis and was also applied in terms of objective. Firstly, a general model of the problem was presented and then completed by describing the required details:

$$\text{MAX } \sum_i E[\min(D_i, Q_i)] p_i - q_i p_i \sum_j l_{ij} m_{ij} \tag{Eq. 1}$$

Subject to:

$$D_i = f(d_i) \tag{Eq. 2}$$

$$d_i = e_i - b_i p_i + b_{ik} P_j \tag{Eq. 3}$$

$$\sum_i q_i (1 - \sum_j l_{ij} - a_i) \leq C \tag{Eq. 4}$$

$$p_1 \geq p_2 \tag{Eq. 5}$$

$$p_i^{\min} \leq p_i \leq p_i^{\max} \tag{Eq. 6}$$

$$d_i \geq 0 \tag{Eq. 7}$$

$$q_i \in Z^+ \tag{Eq. 8}$$

Since it is assumed that the price response function is linear its value may become negative, but negative demand is insignificant in the real world. Thus, the non-negative limitations of the price response function (average demand) were added to the model.

The parameters of the model are defined as follows:

$D_i$  : Total demand for tickets in the i-th class

C: Airplane seat capacity

$d_i$  : Ticket demand in the i-th class (dependent variable)

$\beta_i$  :Absence rate of customers in the i-th class

$m_{ij}$  :Deduction of money returned to the customer in the j-th period for customers in the i-th class

$l_{ij}$  :Ticket cancellation rate in the j-th period for customers in the i-th class

$e_i$  Final demand for tickets (zero ticket price)

$b_i$  The slope of decreased demand

$b_{ik}$  : Demand reduction coefficient to ticket price in another class

$\alpha_i$  : Multiplicative uncertainty parameter in the i-th class at the mean of 1

i: Class ticket (1,2)

j Time intervals

$P_i^{\max}$  Maximum price allowed for the i-th class

$P_i^{\min}$  : Minimum price allowed for the i-th class

$E(.)$  :Expected value function

$F(.)$  :Probability density function

The decision variable of the model is as follows:

$P_i$  : Ticket price in the i-th class

$q_i$  :Number of tickets sold in the i-th class

Since the uncertainty model is expressed as  $D_i = d_i \times \alpha_i$ , so we have:

$$E[\min(D_i, q_i)] = E[\min(d_i \times \alpha_i, q_i)]$$

When  $\alpha_i$  has a uniform distribution in  $[1 - \hat{m}_i, 1 + \hat{m}_i]$ , calculating the Eq. 9 value is as follows. 1

$$E[\min(d_i \times \alpha_i, q_i)] = \int_{-\infty}^{q_i} d_i \alpha_i \frac{1}{2\hat{m}_i} d\alpha_i + \int_{q_i}^{\infty} \frac{q_i}{2\hat{m}_i} d\alpha_i \tag{Eq. 10}$$

By using some calculations, we have:

$$E[\min(d_i \times \alpha_i, q_i)] = \frac{-q_i^2}{4\hat{m}_i d_i} - \frac{d_i(1-\hat{m}_i)^2}{4\hat{m}_i} + \frac{q_i(1+\hat{m}_i)}{2\hat{m}_i} \tag{Eq. 11}$$

Therefore, the model as multiplicative uncertainty by assuming a uniform distribution uncertainty parameter is as follows.

$$\text{Max} \sum_i \left( \frac{-q_i^2}{4\hat{m}_i d_i} - \frac{d_i(1-\hat{m}_i)^2}{4\hat{m}_i} + \frac{q_i(1+\hat{m}_i)}{2\hat{m}_i} \right) P_i - q_i P_i \sum_j l_{ij} m_{ij} \tag{Eq. 12}$$

Subject to

$$d_i = e_i - b_i p_i + b_{ik} P_j \tag{Eq. 13}$$

$$\sum_i q_i (1 - \sum_j l_{ij} - a_i) \leq C \tag{Eq. 14}$$

$$p_1 \geq p_2 \tag{Eq. 15}$$

$$p_i^{\min} \leq p_i \leq p_i^{\max} \tag{Eq. 16}$$

$$d_i \geq 0 \tag{Eq. 17}$$

$$q_i \in Z^+ \tag{Eq. 18}$$

The objective function is non-linear in both models and it is impossible to find the exact optimal solution using conventional mathematical programming methods. As a result, a genetic algorithm is developed for solving these models.

**Findings**

Encoding the search space is the first step to solving a problem using a genetic algorithm. To define the chromosomes, it was stated that each chromosome has four genes while the first and second genes show the prices offered in the first and second classes. In addition, the third and fourth genes represent the number of tickets intended for the two classes.

**Creating the initial population**

After encoding the search space as chromosomes, the first step is creating an initial population for running the genetic algorithm. The initial population quality in terms of optimality and justification leaves a significant effect on the quality of the final solution in the algorithm. The initial population can be created randomly or using a heuristic or meta-heuristic algorithm. In this study, the ticket price was generated randomly in two classes within the reasonable price range for each member of the population to generate the initial population. Then, the corresponding inventory sub-problem was solved by the active-set algorithm of MATLAB to generate the supply values in terms of the randomly generated prices. Then, the resulting optimal values were considered as the supply values for that member. The solutions generated in this way have two critical characteristics:

Firstly, such solutions satisfy the limitations related to airplane capacity and also the limitations related to product prices. These solutions are justified since the upper and lower limits of prices and potential demand in the studied problems are produced in such a way that the amount of product demand is constantly non-negative. Secondly, such solutions are relatively good in terms of the objective function value since the supply values are determined by solving the resulting inventory problem by assuming constant prices.

**Strategies for coping with limitations**

- 1- The upper and lower limits of ticket price in two classes: The strategy of coping with this

group of limitations in the algorithm is combining the elimination strategy and the modification of operators. In the initial population generation phase, the entry of chromosomes which fail to meet such limitations into the initial population was prevented by generating random prices within a reasonable interval. Then, the mutation operator of the algorithm was modified so that it cannot violate this set of limitations and the mutated children satisfy such limitations after applying it to the generated children. By generating the upper and lower limits of prices in a way that the minimum ticket price in the first class is more than the maximum ticket price in the second class, there is no need to make additional modifications in the algorithm to comply with the limitation of the ratio of the ticket prices in the two classes.

2- Airplane capacity limitation: The combination of elimination strategy and penalty function is the strategy of coping with this limitation in the algorithm. In the initial population generation phase, the supply values were generated by solving the inventory problem for random prices. Thus, the resulting supply values satisfy the airplane capacity limitation and prevent the entry of chromosomes violating this limitation into the initial population. While running this algorithm, the solutions which violate the airplane capacity limitation were avoided by adding a penalty function to the value of the objective function.

3- Limitations of supply value non-negativity: The combination of elimination strategy and operator modification strategy is a strategy for coping with this category of limitations. In the initial population generation phase, the supply values were produced by solving the inventory problem for random prices and satisfied the supply values resulting from the non-negativity limitation. Then, the entry of the members violating the non-negativity of supply values to the initial population was prevented. Then, the mutation operator was modified in a way that did not violate this set of limitations. In addition, the mutated children satisfy such limitations after applying them to the generated children.

4- Demand equality limitations: Such limitations are directly replaced in the objective function and there is no need for any extra effort in the algorithm to avoid the solutions violating these limitations.

### **Fitness function**

The roulette wheel selection method was used to select the parents in the mating process. For this purpose, it is necessary to specify the selection probability for each member of the population. This selection probability should be proportional to the fitness of that member. To calculate such probabilities, the researchers divided the fitness of each population member by the total fitness of all members, and the obtained number was determined as the selection probability. Since the objective function of the problem in this study can be negative, the direct calculation of such probabilities from the objective function value can cause some problems. In order to solve this problem at the beginning of each iteration of the algorithm, if there is a negative value between the fitness values of the population members, the ratio of the smallest fitness value is added to all the fitness values, and the resulting numbers are used for calculating the selection probabilities in the roulette wheel method if there is a negative value between the fitness values of the population members.

### **Replacing the new generation**

In the algorithm development, it was assumed that the number of generated children equals the population size in each iteration. In the replacement policy, some members of the previous population plus the generated children are selected adequately based on fitness for transmission to the next generation.



**Intersection operator**

Different intersection operators have been suggested for real-valued genetic algorithms. As the simplest approach, one or more points should be considered in the chromosome as intersection points, and then the values of the variables between such points between the two parents. The final stage of this approach is selecting a certain number of points and making decisions on which parent participates. Such an intersection is called a uniform intersection. It should be noted that no new information is provided in these point intersection methods. Any continuous value in the initial population is transferred to the next generations only in various combinations. Merge-based intersection methods can solve this problem by finding some solutions to merge the variable values from two parents into new variable values in the child. Such methods are as follows:

$$\text{child1} = \beta \times \text{parent1} + (1 - \beta) \times \text{parent2} \tag{Eq. 19}$$

$$\text{child2} = (1 - \beta) \times \text{parent1} + \beta \times \text{parent2} \tag{Eq. 20}$$

In the intersection considered in the genetic algorithm as developed in this study,  $\beta$  is defined as follows.

$$\beta = 0.5 + \text{Normrnd}(0, \frac{1}{15}) \tag{Eq. 21}$$

The linear combination process was used for all variables in this study. Nevertheless, this linear combination process can be applied for any desired number of points. The value of  $\beta$  was considered the same for all genes in this study. In general, this value can be different for different variables.

**Mutation operator**

The approach considered for mutation in this study was adding a normal random number at a mean of zero and a standard deviation equal to the product of the standard deviation of mutation in one-third of the variable value. After that, the mutation operator was modified in a way that if the result is beyond the allowed range of price variables, the price of the intended product can change to the nearest allowed value (high or low price limit). In mathematical terms, the mutation operator can be defined as follows:

$$\text{child}(i) = \text{child}(i) + \text{Normrnd}(0, \text{Mutation SD} \times \frac{\text{child}(i)}{3}) \tag{Eq. 22}$$

**Computational results of genetic algorithm**

This section compares the quality of the genetic algorithm solutions for some sample problems to the quality of the solutions obtained from LINGO to evaluate the efficiency of the genetic algorithm. The used examples are generated randomly for conducting computational experiments. In this regard, 30 examples were generated by using the algorithm, and the results of comparing the genetic algorithm and LINGO were shown in the table. This table presents the ratio of the objective function for the solution obtained from the genetic algorithm to the objective function of the solution obtained from LINGO. In addition, the percentage of the difference between the objective function obtained from the genetic algorithm and the objective function of the solution obtained from LINGO is shown for each problem. As shown in this table, the genetic algorithm has often acted better than LINGO and the values of both indicators confirm this issue.

**Table 1. Comparing the genetic algorithm results with LINGO**

Problem No.	Additive model		Multiplicative model	
	Ratio	Difference	Ratio	Difference
1	1.2467	24.67%	1.0804	8.04%
2	1.2314	23.14%	1.0953	9.53%
3	1.2143	21.43%	1.2765	27.65%
4	1.0649	6.49%	1.1109	11.09%
5	1.2247	22.47%	1.2224	22.24%
6	1.2713	27.13%	1.1237	12.37%
7	1.0912	9.12%	1.0853	8.53%
8	1.2134	21.34%	1.0153	1.53%
9	1.0900	9.00%	1.0747	7.47%
10	1.1401	14.01%	1.2441	24.41%
11	1.1658	16.58%	1.2317	23.17%
12	1.1505	15.05%	1.0488	4.88%
13	1.1760	17.60%	1.0444	4.44%
14	1.2358	23.58%	1.0283	2.83%
15	1.1040	10.40%	0.9984	-0.16%
16	1.1094	10.94%	1.2831	28.31%
17	1.1230	12.30%	1.2002	20.02%
18	1.0238	2.38%	1.0564	5.64%
19	1.2415	24.15%	1.0460	4.60%
20	1.0827	8.27%	1.1853	18.53%
21	1.1678	16.78%	1.2249	22.49%
22	1.2284	22.84%	1.2905	29.05%
23	1.0501	5.01%	1.2310	23.10%
24	1.2671	26.71%	1.0260	2.60%
25	1.1172	11.72%	1.0944	9.44%
26	1.2319	23.19%	1.0868	8.68%
27	1.0357	3.57%	1.0856	8.56%
28	1.0759	7.59%	1.2180	21.80%
29	1.2112	21.12%	1.0505	5.05%
30	1.0593	5.93%	1.1992	19.92%

**Case study**

In this section, the values of parameters for the Tehran-Kish Iran Airtour flight are extracted. In addition, the resulting solution to the problem solved by LINGO and the proposed genetic algorithm is presented.



Table 2 shows the value of parameters defined in the research model for the Tehran-Kish Iran Airtour flight.

**Table 2. The value of parameters defined in the research problem in Iran Airtour**

Row	Variable description	Value
1	Minimum allowed price for class one	180
2	Minimum allowed price for class two	120
3	Maximum allowed price for class one	300
4	Maximum allowed price for class two	230
5	Final demand for class one	500
6	The slope of decreased demand in class one	1.4
7	Reduction factor of demand of class one compared to the price in class two	0.1
8	Final demand for class two	3000
9	The slope of decreased demand in class two	12
10	Reduction factor of demand of class two compared to the price in class one	0.1
11	Absence rate of customers in class one	0.01
12	Absence rate of customers in class two	0.03
13	Ticket cancellation rate in the first period by customers of class one	0.03
14	Deduction of money paid in the first period to customers of class one	0.9
15	Ticket cancellation rate in the second period by customers of class one	0.04
16	Deduction of money paid in the second period to customers of class one	0.7
17	Ticket cancellation rate in the third period by customers of class one	0.02
18	Deduction of money paid in the third period to customers of class one	0.5
19	Ticket cancellation rate in the first period by customers of class two	0.04
20	Deduction of money paid in the first period to customers of class two	0.9
21	Ticket cancellation rate in the second period by customers of class two	0.06
22	Deduction of money paid in the second period to customers of class two	0.7

Row	Variable description	Value
23	Ticket cancellation rate in the third period by customers of class two	0.02
27	Deduction of money paid in the third period to customers of class two	0.5
25	Maximum available capacity	175
26	Multiplicative uncertainty parameter in class one	0.12
27	Multiplicative uncertainty parameter in class two	0.09

Table 2 shows that the minimum and maximum prices offered for tickets in classes one and two are in Thousand Toman. In estimating these prices, the behavior of consumers (customers), their sensitivity to the price, the policies and rates declared by the airline, the company's cost structure, and the ticket final price are all considered. The parameters of the final demand for a specific flight or for the time between two flights are shown in the table. Moreover, the values of these parameters, slopes of decreasing demand, and demand reduction coefficients are estimated by consulting experts. Absence and ticket cancellation rates by passengers in two classes are estimated through the available information within the last year. The information about the deduction of money paid to customers in two classes and different periods, as well as the definition of such periods, is based on the circular developed by the company. The maximum capacity of the airplane is determined based on the type of airplane which is often used on the Tehran-Kish route. The parameters of multiplicative uncertainty are realized by asking experts for opinions.

The price and number of tickets offered in two classes in the final solution are described in Table 3 by formulating the problem using the above-mentioned data and solving by LINGO:

**Table 3. The optimal solution obtained from LINGO**

Class	Optimal price	The optimal number of tickets
One	278	35
Two	143	140

The final value of the objective function equals 22176. In addition, the value of the objective function is in thousand Toman. Furthermore, we can have the final solution about the price and number of tickets offered in two classes by applying the genetic algorithm to solve the above-mentioned problem.

**Table 4. The optimal solution obtained from the genetic algorithm**

Class	Optimal price	The optimal number of tickets
One	264	42
Two	157	133

The final value of the objective function equals 25824, which is in thousand Toman. Based on the comparison of the above tables, the final value of the objective function in the solution, found by the developed genetic algorithm compared to the solution found by LINGO, shows an improvement of more than 16%, which is an interesting figure indicating the highly appropriate efficiency of the developed algorithm for solving the model. Furthermore, this is the same result obtained by the numerical tests. Comparing the value of the decision variables in the two solutions indicates that the price of tickets in class one and the number of tickets offered in class two decrease while the price of tickets in class two and the number of tickets offered in class one decrease in the solution provided by the genetic algorithm compared to LINGO.

### **Conclusion**

This study considered a problem in optimizing the price of plane tickets regarding the demand uncertainty, the cancellation rate of tickets, the absence rate, and the classification of plane tickets. In addition, this study developed a mathematical model related to the problem. Genetic algorithm and Lingo were used to solve the model since it was a mixed integer non-linear mathematical programming and its objective function is non-concave. It should be noted that the optimal ticket prices were obtained by considering the limitations raised in the model. Eventually, a comparison was made between the results of the genetic algorithm compared to Lingo. As a result, the results related to the genetic algorithm were more satisfactory. According to the observations and results related to a sample of 30 random problems, it was realized that the genetic algorithm developed in this study could be appropriate for solving the model in the real world and with actual data. The models of finding optimal prices for tickets (products) will be non-concave nonlinear models if the capacity or inventory is considered in them as well. Heuristic or meta-heuristic algorithms are the only solutions for solving such models. As shown in many studies, genetic algorithms have good efficiency for solving non-convex non-linear problems, confirming the evaluation of the algorithm efficiency developed in this study. According to the high efficiency of the proposed genetic algorithm for solving the developed problem based on the data of Iran Airtour, the suitability of the proposed algorithm is true not only in hypothetical problems but also in real-world problems. This study assumed that the non-deterministic component of the demand function is random. Considering the parameter as fuzzy can be an appropriate development for this model. Furthermore, regarding the demand pattern as non-deterministic and using other distribution functions such as Poisson distribution can be considered an appropriate development for this model. Nevertheless, the absence and cancellation rate can be considered uncertain.

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